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(S, d) Magic Labeling Of Non-Unicyclic Graphs-Paper-I

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Abstract. *Let G (p, q) be a connected, undirected, simple and non-trivial graph with p vertices and q edges. Let f be an injective function f: V(G) → { $s, s+d, s+2d, \ldots s+(q+1)d$ } and g be an injective function g: E(G) → $\{d, 2d, 3d \ldots 2(q-1)d\}$.Then the function f is said to be (s, d) magic labeling if $f(u) + g(uv) + f(v)$ is a constant, for all $u, v \in V(G)$ and $uv \in E(G)$. A graph G is called (s, d) magic graph if it admits (s, d) magic labeling.*

Keywords: *Mirror graph, Jewel graph and Helm graph*

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I. INTRODUCTION

Graphs considered here are finite and simple. Numerous fields of science and technology, including astronomy, circuit design, coding theory, etc., use graph labeling. A vast body of literature has been produced on the subject, with almost 1300 papers published. They give rise to families of graphs with appealing names like as prime labeling, magic, antimagic, bi magic, harmonious, felicitous, elegant, cordial, and so on. Sedlácek was the one who initially presented the idea of Magic labeling. J. Jayapriya and K. Thirusangu demonstrated the existence of labeling for a certain class of graphs and presented 0-Edge Magic Labeling. We introduce (s, d) Magic labeling of graphs. If G admits (s, d) Magic labeling, then G is called as (s, d) Magic graph. In this paper, a new concept of (s, d) Magic labeling has been introduced for some graphs.

[5] Let $G\ (p, q)$ be a simple, non-trivial, connected, undirected graph with p vertices and q edges. Consider the following: $f: V(G) \to \{s, s+d, s+2d, ..s+(q+1)d\}$ and
$g: E(G) \to \{d, 2d, 3d \ldots 2(q-1)d\}$ be an injective function. Then, for any $u, v \in V(G)$ and $uv \in E(G)$, $f(u) + g(uv) + f(v)$ is a constant, and the function f is said to be (s, d) magic labeling. If a graph G admits (s, d) magic labeling, then it is referred to as a (s, d) magic graph.

II. DEFINITIONS

Definition 2.1: Let G be a graph. Let G' be a copy of G . The Mirror graph M(G) of G is defined as the disjoint union of G and G' with additional edges joining each vertex of G to its corresponding vertex in G'.

Definition 2.2: The Jewel graph J_n is a graph with the vertex set V (J_n)= $\{u, v, x, y, u_i : 1 \leq i \leq n\}$ and the edge set $E(J_n)$= $\{ux, uy, xy, xv, yu, uu, vu : 1 \leq i \leq n\}$.
Definition 2.3: Helms H_n(graph obtained from a wheel by attaching a pendent edge at each vertex of the n – cycle)

III. Main Result

Theorem 3.1 The Mirror graph $M(P_\eta)$ is (s, d) magic labeling.
Proof: Let G be a mirror graph $M(P_\eta)$,the vertex set be $V(M(P_\eta))$={ $v_i, v_i' : 1 \leq i \leq \eta$}
$E(M(P_\eta))$={ $v_i v_i' : 1 \leq i \leq \eta\} \cup \{v_i v_{i+1} : 1 \leq i \leq \eta - 1\} \cup \{v_i' v_{i+1}' : 1 \leq i \leq \eta - 1\}$
Define the function f from the vertex set to $\{s, s+d, s+2d \ldots .s+(q+1)\}$,
$g: E(G) \to \{d, 2d, 3d \ldots ..2(q-1)d\ \}$

Case(a) When η is Odd

Labeling of vertices		
Value of i	$f(v_{i+1})$	$f(v'_{i+1})$
$0 \le i \le \eta - 1$	$s + 2id$	$s + (2i+1)d$

Table 1 Labeling of Vertices of the graph $M(P_\eta)$

Labeling of Edges			
Value of i	$g(v_i v_{i+1})$	$g(v_i v'_i)$	$g(v'_i v'_{i+1})$
$1 \le i \le \eta$	-	$2s + 2(q-1)d - (f(v_i) + f(v'_i))$	-
$1 \le i \le \eta - 1$	$2s + 2(q-1)d - (f(v_i) + f(v_{i+1}))$	-	$2s + 2(q-1)d - (f(v'_i) + f(v'_{i+1}))$

Table 2 Labeling of edges of the graph $M(P_\eta)$

Case(b) When η is even

Labeling of vertices		
Value of i	$f(v_{i+1})$	$f(v'_{i+1})$
$0 \le i \le \eta - 1$	$s + id$	$s + (\eta + i)d$

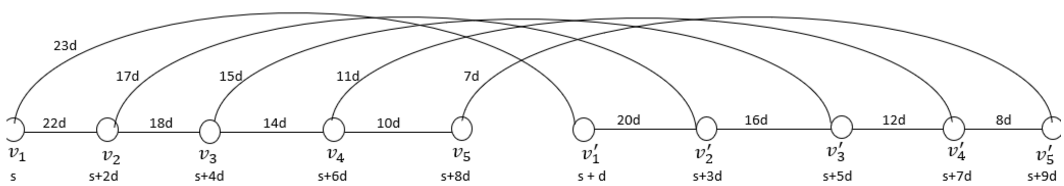
Table 3 Labeling of Vertices of the graph $M(P_\eta)$

Labeling of Edges			
Value of i	$g(v_i v_{i+1})$	$g(v_i v'_i)$	$g(v'_i v'_{i+1})$
$1 \le i \le \eta$	-	$2s + 2(q-1)d - (f(v_i) + f(v'_i))$	-
$1 \le i \le \eta - 1$	$2s + 2(q-1)d - (f(v_i) + f(v_{i+1}))$	-	$2s + 2(q-1)d - (f(v'_i) + f(v'_{i+1}))$

Table 4 Labeling of edges of the graph $M(P_\eta)$

Therefore $f(v_i) + f(v_{i+1}) + g(v_i v_{i+1}), f(v_i) + f(v'_i) + g(v_i v'_i), f(v'_i) + f(v'_{i+1}) + g(v'_i v'_{i+1})$ are constant equals to $2(s + (q-1)d)$.Hence the mirror graph $M(P_\eta)$ admits (s, d) magic labeling

Mirror graph M(P_5)



Theorem 3.2 The Jewel graph J_η is (s, d) magic labeling

Proof: Let J_η be the Jewel graph. $|V(J_\eta)| = \eta + 4$ $|E(J_\eta)|$= 2η+5.
Let $V(J_\eta) = \{u, v, x, y, u_i\, ; 1 \le i \le \eta\}$ and $E(J_\eta) = \{xu, uy, vy, vx, uu_{i,}, vu_{i,}: 1 \le i \le \eta\}$
Define the function f from the vertex set to $\{s, s + d, s + 2d \ldots . s + (q + 1)\}$,
$g: E(G) \to \{d, 2d, 3d \ldots . .2(q - 1)d \;\; \}$

Labeling of vertices	
$f(x) = s$	
$f(y) = s + d$	
$f(u) = s + 2(\eta + 2)d$	
$f(v) = s + (\eta + 2)d$	
Value i	$f(u_i)$
$1 \le i \le \eta$	$s + (i + 1)d$

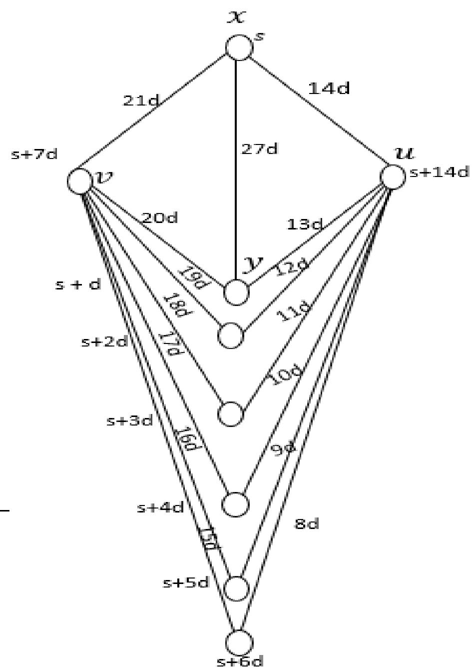
Table 5: Labeling of vertices of the graph J_η

Labeling of edges		
$g(ux) = 2s + 2(q - 1)d - (f(u) + f(x))$		
$g(uy) = 2s + 2(q - 1)d - (f(u) + f(y))$		
$g(vy) = 2s + 2(q - 1)d - (f(v) + f(y))$		
$g(vx) = 2s + 2(q - 1)d - (f(v) + f(x))$		
Value i	$g(uu_i)$	$g(vv_i)$
$1 \le i \le \eta$	$2s + 2(q - 1)d - (f(u) + f(u_i))$	$2s + 2(q - 1)d - (f(v) + f(v_i))$

Table 6: Labeling of edges of the graph J_η

Therefore $f(u) + f(u_i) + g(uu_i), f(v) + f(v_i) + g(vv_i)$, $f(u) + f(x) + g(ux)$, $f(u) + f(y) + g(uy)$, $f(v) + f(y) + g(vy)$, $f(v) + f(x) + g(vx)$ are constant equals to $2(s + (q - 1)d)$.Hence the Jewel graph J_η admits (s, d) magic labeling.

Jewel graph J_5



Theorem 3.3 The Helm graph H_η is (s,d) magic labeling.

Proof: Let graph H_η have a vertex set $V(H_\eta)=\{v, u_1, u_2 \ldots u_\eta, v_1, v_1, \ldots v_\eta\}$

$E(H_\eta)=\{vu_i : i = 1,2..\eta\} \cup \{u_i v_i : i = 1,2..\eta\} \cup \{u_i u_{i+1} : i = 1,2 \ldots \eta - 1\} \cup \{u_\eta u_1\}$

Here $|V(H_\eta)| = 2\eta + 1$, $|E(H_\eta)| = 3\eta$

Define the function f from the vertex set to $\{s, s+d, s+2d \ldots . s+(q+1)\}$,

$g: E(G) \to \{d, 2d, 3d \ldots . 2(q-1)d\}$

Case(a) When η is even

Labeling of vertices						
Value of i	$f(u_{i+1})$	$f(u_{2i})$	$f(u_{2i+1})$	$f(v_i)$	$f(v_{2i})$	$f(v_{2i+1})$
$i = 0$	s	-	-	-	-	-
$i = 1$	$s+d$	-	-	$s+\eta d$	-	-
$2 \le i \le \frac{\eta}{2}$	-	$s+2(i-1)d$	-	-	-	-
$1 \le i \le \frac{\eta}{2} - 1$	-	-	$s+(\eta+2i)d$	-	-	$s+2id$
$1 \le i \le \frac{\eta}{2}$	-	-	-	-	$s+(\eta+(2i-1)d$	--

Table 7: Labeling of vertices of the graph H_η

Labeling of Edges				
Value of i	$g(u_i v_i)$	$g(u_i u_{i+1})$	$g(u_i u_1)$	$g(vu_i)$
$1 \le i \le \eta$	$2s+2(q-1)d - (f(u_i)+f(v_i))$	-	-	-
$1 \le i \le \eta - 1$	-	$2s+2(q-1)d - (f(u_i)+f(u_{i+1}))$	-	$2s+2(q-1)d-(f(u_i) + f(v))$
$i = \eta$	-	-	$2s+2(q-1)d - (f(u_1)+f(u_i))$	

Table 8: Labeling of edges of the graph H_η

Case(b) When η is Odd

Labeling of vertices						
Value of i	$f(u_{i+1})$	$f(u_{2i})$	$f(u_{2i+1})$	$f(v_i)$	$f(v_{2i})$	$f(v_{2i+1})$
$i = 0$	$s+\eta d$	-	--	--	-	-

$i = 1$	-	-	-	s	-	-
$1 \leq i \leq \frac{\eta - 1}{2}$	-	$s + 2(i-1)d$	$s + (\eta + 2i)d$	-	$s + (\eta + (2i-1)d$	$s + 2id$

Table 9: Labeling of vertices of the graph H_η

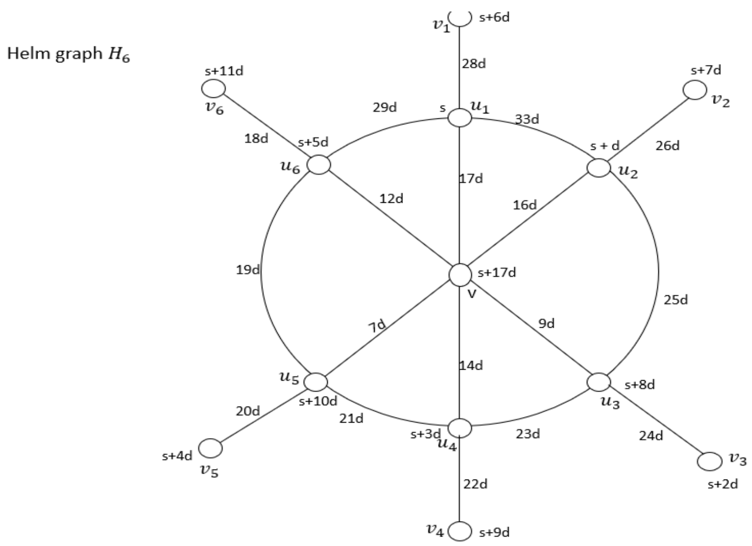
Labeling of edges				
Value of i	$g(u_i v_i)$	$g(u_i u_{i+1})$	$g(u_i u_1)$	$g(v u_i)$
$1 \leq i \leq \eta$	$2s + 2(q-1)d - (f(u_i) + f(v_i))$	-	-	-
$1 \leq i \leq \eta - 1$	-	$2s + 2(q-1)d - (f(u_i) + f(u_{i+1}))$	-	$2s + 2(q-1)d - (f(u_i) + f(v))$
$i = \eta$	-	-	$2s + 2(q-1)d - (f(u_1) + f(u_i))$	-

Table 10: Labeling of edges of the graph H_η

Therefore

$f(u_i) + f(v_i) + g(u_i v_i), f(u_i) + f(u_{i+1}) + g(u_i u_{i+1}), f(u_i) + f(u_1) + g(u_i u_1), f(u_i) + f(v) + g(u_i v)$

are constant equals to $2(s + (q-1)d)$.Hence the helm graph H_η admits (s, d) magic labeling



IV. Conclusions

In this study, a (s, d) Magic Labeling has been discovered for a few graphs. Future research will examine the (s, d) Magic labeling of additional graphs and some graph families.

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